

Tema 1
EDO(1) NL

$$M(x, y) + N(x, y) \frac{dy}{dx} = 0$$

Método de Variables Separables

$$P(x)Q(y) + R(x)S(y) \frac{dy}{dx} = 0$$

(SG) $\int \frac{P(x)}{R(x)} dx + \int \frac{S(y)}{Q(y)} dy = C$

(1) $y + x \frac{dy}{dx} = 0$

$$P(x) = 1 \quad Q(y) = y$$

$$R(x) = x \quad S(y) = 1$$

(SG) $\int \frac{1}{x} dx + \int \frac{1}{y} dy = C$

$$L(x) + \int \frac{dy}{Ly} = C$$

$$L(x) + L(Ly) = C$$

Método de Coeficientes Homogéneos

$$M(x, y) + N(x, y) \frac{dy}{dx} = 0$$

$$\lambda = \text{cte.} \quad \begin{aligned} M(\lambda x, \lambda y) &= \lambda^m M(x, y) \\ N(\lambda x, \lambda y) &= \lambda^n N(x, y) \quad m=n \end{aligned}$$

$$x \frac{dy}{dx} = \sqrt{x^2 - y^2} + y \quad (CH)$$

$$-(\sqrt{x^2 - y^2} + y) + x \frac{dy}{dx} = 0$$

$$M(\lambda x, \lambda y) = -(\sqrt{(\lambda x)^2 - (\lambda y)^2} + \lambda y)$$

$$= -(\sqrt{\lambda^2 x^2 - \lambda^2 y^2} + \lambda y)$$

$$= -(\sqrt{\lambda^2 (x^2 - y^2)} + \lambda y)$$

$$= -(\sqrt{\lambda^2} \sqrt{x^2 - y^2} + \lambda y)$$

$$= -(\lambda \sqrt{x^2 - y^2} + \lambda y)$$

$$= -\lambda (\sqrt{x^2 - y^2} + y) \quad m=1$$

$$N(\lambda x, \lambda y) = \lambda x \quad n=1$$

$$-(\sqrt{x^2 - y^2} + y) + x \frac{dy}{dx} = 0$$

Substitución

$$y(x) = x \cdot u(x) \quad y'(x) = x \cdot u'(x) + u(x)$$

$$u(x) = \frac{y(x)}{x}$$

$$-(\sqrt{x^2 - (x \cdot u(x))^2} + x \cdot u(x)) + x (x \cdot u'(x) + u(x)) = 0$$

$$-(\sqrt{x^2 - x^2 u^2(x)} + x \cdot u(x)) + x^2 u'(x) + x u(x) = 0$$

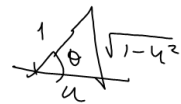
$$-\sqrt{x^2} \sqrt{1 - u^2(x)} + x^2 u'(x) = 0$$

$$-x \sqrt{1 - u^2(x)} + x^2 u'(x) = 0$$

$$P(x) \quad Q(u) \quad R(x) \quad S(u) = 1$$

$$\int \frac{-x}{x^2} dx + \int \frac{1}{\sqrt{1-u^2}} du = C$$

$$-\int \frac{dx}{x} + \int \frac{du}{\sqrt{1-u^2}} = C$$



$$\cos(\theta) = \frac{u}{1}$$

$$\sin(\theta) = \frac{\sqrt{1-u^2}}{1}$$

$$u = \cos(\theta) \quad du = -\sin(\theta) d\theta$$

$$\sqrt{1-u^2} = \sin(\theta)$$

$$+ \int \frac{-\sin(\theta) d\theta}{\sin(\theta)} = -\int d\theta$$

$$\theta = \arccos(u) = -\theta$$

$$-\ln x - \arccos(u) = C$$

$$-\ln x - \arccos\left(\frac{y(x)}{x}\right) = C$$

$$4x^2 - xy + y^2 + (x^2 - xy + 4y^2) \frac{dy}{dx} = 0$$

$$y = x \cdot u \quad \frac{dy}{dx} = x \frac{du}{dx} + u$$

$$4x^2 - x(xu) + (xu)^2 + (x^2 - x^2u + 4x^2u^2) \left(x \frac{du}{dx} + u \right) = 0$$

$$4x^2 - x^2u + x^2u^2 + x^3 \frac{du}{dx} - x^3u \frac{du}{dx} + 4x^3u^2 \frac{du}{dx} +$$

$$x^2 \left(4 - \cancel{u} + \cancel{u^2} + \cancel{u} - \cancel{u^2} + 4u^3 \right) + x^3 \left(\frac{du}{dx} - u \frac{du}{dx} + 4u^2 \frac{du}{dx} \right) = 0$$

$$(4 + 4u^3) + x(1 - u + 4u^2) \frac{du}{dx} = 0$$

$$P=1 \quad Q=4+4u^3 \quad R=x \quad S=(1-u+4u^2)$$

$$\int \frac{1}{x} dx + \int \frac{1-u+4u^2}{4+4u^3} du = C$$

$$\log(x) + \frac{\log(u+1)}{2} + \frac{\log(u^2-u+1)}{2} = C$$

$$\log(x) + \frac{\log\left(\frac{y}{x}+1\right)}{2} + \frac{\log\left(\left(\frac{y}{x}\right)^2 - \frac{y}{x} + 1\right)}{2} = C$$